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Briggs's Method of Interpolation; being a translation of the 13th Chapter and part of the 12th of the Preface to the "Arithmetica Logarithmica," by* J. HILL WILLIAMS, *Esq., one of the Vice-Presidents of the Institute of Actuaries.*

[Read before the Institute, 30th December, 1867.]

THOSE of our readers who have studied the paper of M. Maurice on Interpolation, of which a translation appeared in our last number, will no doubt be glad to compare with it Briggs's own description of his method of Interpolation. His original work however appears to be very scarce; and the chapters in which he describes his method—the 12th and 13th—are omitted, even in the Edition published by Vlacq in Briggs's lifetime. We believe, therefore, that this translation by Mr. Williams of those parts of Briggs's Preface in which he describes his method of Interpolation, will prove very acceptable to our readers.—ED. *J. I. A.*

CHAPTER XII.

Given two consecutive integers and their Logarithms: it is required to interpolate between them nine other equidistant numbers, and to find their Logarithms.

If the second differences of the given Logarithms are nearly equal, this will be an easy matter: but if the third differences cannot be neglected, this method will be found somewhat defective.

* Briggs, like most, if not all, of his contemporaries, wrote in Latin.

Take two consecutive numbers A, and their Logarithms B, together with their first differences C, and their second differences D. If the second differences are equal, multiply either of them into the numbers standing opposite the first ten natural numbers in the subjoined Table E; then, the three last figures having been cut off each of the products F, G, H, I, K, the first five are to be added to the tenth part of the first difference of the two given logarithms, and the last five are to be subtracted from the same. The sums and the remainders will be the differences of the Logarithms sought; and the successive addition of these differences to the smaller of the given Logarithms, will give the Logarithms required. For example, let the given numbers be 91235 and 91236, the first difference of their logarithms being 47601,4799.

	47602,0016.C	
91235.A.	4·96016,14763,8639.B	5217.D
	47601,4799.C	
91236.A.	4·96016,62365,3438.B	5217.D
	47600,9582.C	

TABLE E.		
1	45	Products to be added.
2	35	
3	25	
4	15	
5	5	
6	5	Products to be subtracted.
7	15	
8	25	
9	35	
10	45	

Natural numbers.	Logarithms.	
912350	4·96016,14763,8639	
	4760,1715	C+F
1	4·96016,19524,0354	
	4760,1662	C+G
2	4·96016,24284,2016	
	4760,1610	C+H
3	4·96016,29044,3626	
	4760,1558	C+I
4	4·96016,33804,5184	
	4760,1506	C+K
912355	4·96016,38564,6690	
	4760,1454	C−K
6	4·96016,43324,8144	
	4760,1402	C−I
7	4·96016,48084,9546	
	4760,1350	C−H
8	4·96016,52845,0896	
	4760,1297	C−G
9	4·96016,57605,2193	
	4760,1245	C−F
912360	4·96016,62365,3438	

Products.		5217. Multiplicand.	
F ..234	765	45	Multipliers.
G ..182	595	35	
H ..130	425	25	
I .. 78	255	15	
K .. 26	085	5	
47601479	9	$\frac{1}{10}$C	
47601714	7.	C+F	
47601662	5.	C+G	
47601610	3.	C+H	
47601558	2.	C+I	
47601506	0.	C+K	
47601453	8.	C−K	
47601401	6.	C−I	
47601349	5.	C−H	
47601297	3.	C−G	
47601245	1.	C−F	

If the second differences are unequal, as below:* add the two consecutive second differences, take half the sum for the second difference, and multiply as before.

	4,51707,8187 C	
9615.A	3·98294,92885,7450 B	469771 D*
	4,51660,8416 C	
9616.A	3·98299,44546,5866 B	469672 D
	4,51613,8744 C	
		939443 Sum
		469721 ½ Sum

Products.		469721 Multiplicand
F. 21137	445	45
G. 16440	235	35
H. 11743	025	25
I. . .7,045	815	15
K. . 2348	605	5

Multipliers

96150	3·98294,92885,7450	¶
	45168,1979	C+F
1	3·98295,38053,9429	
	45167,7282	C+G
2	3·98295,83221,6711	
	45167,2585	C+H
3	3·98296,28388,9296	
	45166,7887	C+I
4	3·98296,73555,7183	
	45166,3190	C+K
96155	3·98297,18722,0373	
	45165,8493	C−K
6	3·98297,63887,8866	
	45165,3796	C−I
7	3·98298,09053,2662	**
	45164,9099	C−H†
8	3·98298,54218,1761	
	45164,4401	C−G
9	3·98298,99382,6162	
	45163,9704	C−F
96160	3·98299,44546,5866	

451660841	6	$\frac{1}{10}$C
451681979	0	C+F
77281	8	C+G
72584	6	C+H
67887	4	C+I
63190	2	C+K
451658493	0	C−K
53795	8	C−I
49098	6	C−H
44401	4	C−G
39704	2	C−F

469721
105

2348605
469721

49320|705 §

4516608416
7

Products { 3161625891|2
49320|7 §

3161675212
¶ 398294928857450

** 96157 398298090532662

451660841|6 $\frac{1}{10}$C
Product. . 11743|0

† Remainder 4516490986|6

Table E′.

1	45	Products to be added
2	80	
3	105	
4	120	
5	125	
6	120	
7	105	
8	80	
9	45	

But, suppose you wish to find any one of the logarithms without the others. Multiply the number less than ten which is written at the end of the given number A, into the given difference C; multiply also the number standing opposite to it in Table E′ into the second difference; and, cutting off three figures from the latter

product, and one from the former, add the products: the sum added to the given Logarithm will give the Logarithm sought. If, for example, you wish to know what is the Logarithm of the number 96157, the process is as follows. The given difference 4516608416 is to be multiplied by 7. The product is 31616258912. Then taking 105, the number opposite to 7 in Table E′, multiply it into the second difference 469721; add the product 49320 | 705 (with the three last figures cut off) to the first product (with one figure cut off) 3161625891. Add the total 3161675212 to the given Logarithm ¶, and the total, 398298090532662, will be the required Logarithm of the number 96157. If you wish to know the difference between the Logarithm of this number and that of the next higher number: multiply the number in the Table standing opposite the number 8, which is greater by unity than the given 7, into the second difference 469721; subtract the product 11743|025 (with three figures cut off) from the tenth part of the given first difference, and the remainder 451649099 will be the difference sought.†

[The remainder of this Chapter describes the method of finding the number corresponding to a given Logarithm. It would not be intelligible without quotations from former Chapters; and as it does not illustrate the direct method of interpolation, it is here omitted.]

CHAPTER XIII.

To find the Logarithms of the omitted Thousands of natural numbers [20,000 to 90,000, not calculated in his Tables]; or, given any equidistant numbers whatsoever, together with their Logarithms, to find the Logarithms of the four numbers interpolated at equal intervals between each adjacent two.

The intermediate Logarithms may be obtained in various ways. I think the following is the best way; the others we will consider afterwards.

Take the first, second, third, fourth and other differences of the given Logarithms; and divide the first differences by 5, the second by 25, the third by 125, and so on; the divisors increasing in a quintuple ratio; and call the quotients the first, second, third, &c., *mean* differences. Or, instead of dividing, multiply the first given differences by 2, the second by 4, the third by 8, and so on; cutting off in the products, one figure from the first product, two from the next, three from the third, and so on: [*i.e.* multiply the

given differences respectively by ·2, by ·04, by ·008, &c.] These products (which are equal to the quotients above-described) will be the first, second, third, &c., mean differences. For example, let the following Logarithms be given, together with their first, second, third, fourth and fifth differences, which in fact are found from the given Logarithms by subtraction.

Differences.					Logarithms.	Natural Nos.
Fifth.	Fourth.	Third.	Second.	First.		
					33232,52100,17169	2105
				103035512600		
			*243871263		33242,82455,29769	2110
		1151695		102791641337		
	8138		242719568		33253,10371,71106	2115
75		1143557				
				102548921769		
	8063		241576011		33263,35860,92875	2120
75		1135494		102307345758		
	7988		240440517		33273,58934,38633	2125
		1127506		102066905241		
			239313011		33283,79603,43874	2130
				101827592230		
					33293,97879,36104	2135

We have next to find the mean differences. Multiplying the given first differences by 2, and cutting off the last figure, we get the first mean differences. The remaining mean differences will be found by multiplying the other differences by 4, 8, 16, 32, &c., and cutting off 2, 3, 4, 5, ... figures from the products.

Then these mean differences are to be corrected in the following manner:

The two highest differences—the fifth and the fourth—cannot be corrected, because the seventh and sixth are nothing: for every correction of the differences is made by subtracting the alternate corrected differences of the higher orders: thus, the subtraction of the seventh differences corrects the fifth: that of the sixth, corrects the fourth, &c. Therefore in the present case, the fourth and fifth *mean* differences are taken for the fourth and fifth *corrected* differences.

Every third mean difference however is corrected by subtracting from it three times the fifth corrected difference.

* The numbers here printed in antique type are not inserted in the original; but they have been added to make the author's process more easily followed, in conformity with his remark made further on, p. 81.

	Mean Differences.		
First.	20558328267	4	
	20509784353	8	× ·2
	20461469151	6	
	20413381048	2	
Second	9708782	72	
	9663040	44	× ·04
	9617620	68	
Third	9213	560	
	9148	456	× ·008
	9083	952	
	9020	048	
E Fourth	13	0208	
	12	9008	× ·0016
	12	7808	
Fifth		02400	× ·00032
		02400	

9213	560	third mean difference	
	72	three times the fifth corrected difference	
9213	488	third corrected difference	
9148	456	third mean ,,	
	72	three times the fifth corrected ,,	
9148	384	third corrected difference	D
9083	952	third mean ,,	
	72		
9063	880	third corrected ,,	
9020	048		
	72		
9019	976	third corrected ,,	

From the second mean difference we must subtract twice the fourth corrected difference—and we must moreover take $\frac{7}{5}$ ($1\frac{2}{5}$) of the sixth difference, if any sixth differences have been found in the work.

Third corrected	9213	5	
	9148	4	D
	9083	9	
	9020	0	
Second corrected	9708756	7	
	9663014	6	C
	9617595	1	
First corrected	20558319053	9	
	20509775205	4	B
	20461460067	7	
	20413372020	2	

9708782	72	second mean difference	
26	04	twice the fourth corrected difference	
9708756	68	second corrected difference	
9663040	44	second mean ,,	
25	80	twice the fourth corrected ,,	C
9663014	64	second corrected difference	
9617620	68		
25	56	twice the fourth corrected ,,	
9617595	12	second corrected difference	

From each first mean difference we must deduct the corresponding third corrected difference and $\frac{1}{5}$ of the fifth difference.

20558328267	4 first mean difference	$\frac{1}{5}$ of the fifth falls outside the limits and may therefore be safely neglected.
9213	5 third corrected difference	
	0048 $\frac{1}{5}$ of the fifth ,,	
20558319053	9 first corrected ,,	
20509784353	8 first mean ,,	
9148	4 third corrected ,,	
20509775205	4 first corrected ,,	B
20461169151	6	
9083	9 third corrected ,,	
20461460067	7 first corrected ,,	
20413381048	2	
9020	0	
20413372028	2 first corrected ,,	

In this manner then have all the differences been corrected and prepared for use. If there were more orders of differences, we should proceed in the same way, commencing with the highest orders, which we always suppose to be the least.

The following Table shows what multiple of each difference is to be subtracted in each case:

TABLE X.

A	B	C	D	E	F	G	H	I
20								
19								
18	18(20)							
17	17(19)							
16	16(18)	123·2(20)						
15	15(17)	108·0(19)						
14	14(16)	93·8(18)	400·4(20)					
13	13(15)	80·6(17)	317·2(19)					
12	12(14)	68·4(16)	246·4(18)	629·64(20)				
11	11(13)	57·2(15)	187·0(17)	431·20(19)				
10	10(12)	47·0(14)	138·0(16)	283·80(18)	434·40(20)			
9	9(11)	37·8(13)	98·4(15)	177·84(17)	236·88(19)			
8	8(10)	29·6(12)	67·2(14)	104·72(16)	118·72(18)	111·248(20)		
7	7 (9)	22·4(11)	43·4(13)	56·84(15)	53·20(17)	36·680(19)		
6	6 (8)	16·2(10)	26·0(12)	27·60(14)	20·40(16)	10.760(18)	4·080(20)	
5	5 (7)	11·0 (9)	14·0(11)	11·40(13)	6·20(15)	2·280(17)	·500(19)	
4	4 (6)	6·8 (8)	6·4(10)	3·64(12)	1·28(14)	·272(16)	·032(18)	·0016(20)
3	3 (5)	3·6 (7)	2·2 (9)	·72(11)	·12(13)	·008(15)		
2	2 (4)	1·4 (6)	·4 (8)	·04(10)				
1	1 (3)	·2 (5)						

The numbers placed in column A denote the mean differences of the first, second, third and other orders up to the 20th. But the numbers in the columns B, C, D, &c., show what multiples of each corrected difference are to be subtracted* from those mean differences which are placed in column A in the same line with them. For example: from the sixth mean difference we must subtract six times the eighth corrected difference; $16\frac{2}{10}$ of the 10th corrected difference; 26 times the 12th corrected difference; and so on; and in the same manner from the first mean difference we must subtract the third corrected difference and $\frac{1}{5}$ of the fifth difference.

* Not only for Logarithms are all to be subtracted, but also for Tangents, Secants, and for any the same powers of equidistant numbers. For Sines, however, the differences contained in columns B, D, F, H, are to be added to the mean differences placed in column A: but the others in columns C, E, G, I, are to be subtracted.

Having found these corrected differences, the next step will be to insert each conveniently in its place, in order that in so complicated an operation all confusion may as far as possible be avoided. We shall accomplish this more readily if we have a sheet of cross-ruled paper divided as in the following Table, and if the first, third, fifth, seventh, and other odd differences are written in a different coloured ink from the others.* The given Logarithms marked A occupy every fifth place. The second corrected differences, C, the fourth, E, the sixth, the eighth, &c., are placed to the left in the same line as the Logarithms. But the first corrected differences, B, the third, D, the fifth, seventh, &c., are placed in the centre of each space. Lastly, the vacant places are to be filled up, beginning from the left. By the addition of the fourth differences, we obtain the third: by the addition of the third, we obtain the second; and so on: and in the process of addition we may either add or subtract a unit in the last place, as required. For with irrational quantities, it will be sufficient to have differences approximately true, since we cannot find the true values exactly. For this reason, although I said in the beginning of this chapter that the last figure was to be cut off from the products of the first differences by 2, yet here I have cut off none; but, in the first and remaining differences, I have thought it better to retain one figure beyond the established limits, in order that the work may proceed with greater certainty of accuracy. I recommend the same course to be pursued with Tangents, Secants, and Sines; but in dealing with the powers of equidistant numbers, where the given numbers and all the differences are rational, all may be contained within the prescribed limits; for there always exists a definite number of orders of differences, which cannot be exceeded, when the difference between the numbers is constant. For instance, in squares there are two orders of differences; in cubes, three orders; in the fourth powers, four orders; and so on. And the differences of the highest order are always equal to each other, and equal to the product of the same power of the common difference into the continued product of the index of that power into all lower numbers [equal to $n(n-1)(n-2)\ldots 2.1.b^n$ if the numbers are a^n, $(a+b)^n$, $(a+2b)^n \ldots$] so that if the difference of the given numbers is 1, the last differences will be in the case of squares, 2; in cubes, 6; in the fourth powers, 24; in (5), 120; in (6), 720; in (7), 5040; &c.; these numbers being the continuous products $1.2=2$, $1.2.3=6$, $1.2.3.4=24$, &c. But if the difference of the given numbers be 3, the difference of

* These differences are here printed in antique type.

the highest order will be in the case of the squares 18,—the product of the square 9 into 2; in the case of the cubes, 162,—the product of the cube 27 into 6; in the case of the fourth powers, 1944,—the product of the fourth power 81 into 24, &c.

4th Differences.		2nd and 3rd Differences.					Logarithms and 1st Differences.					Natural numbers.
				9213	5	D	2	05583	19053	9	B	
			97	27144	5							
				9200	4		2	05485	91909	5		
			97	17944	1							
				9187	4		2	05388	73965	4		
E	13	0	97	08756	7	C	33253	10371	71106		A	2115
				9174	4		2	05291	65208	7		
			96	99582	4		255	15663	36315			16
				9161	4		2	05194	65626	3		
			96	90421	0		257	20858	01941			17
				9148	4	D	2	05097	75205	4	B	
			96	81272	6		259	25955	77146			18
				9135	5		2	05000	93932	7		
			96	72137	1		261	30956	71079			19
				9122	6		2	04904	21795	6		
E	12	9	96	63014	6	C	33263	35860	92875		A	2120
				9109	7		2	04807	58781	0		
			96	53905	0		265	40668	51656			21
				9096	8		2	04711	04876	0		
			96	44808	2		267	45379	56532			22
				9083	9	D	2	04614	60067	7	B	
			96	35724	4		269	49994	16600			23
				9071	1		2	04518	24343	2		
			96	26653	3		271	54512	40943			24
				9058	3		2	04421	97689	9		
E	12	8	96	17595	1	C	33273	58934	38633		A	2125
				9045	5		2	04325	80094	8		
			96	08549	6							
				9032	7		2	04229	71545	2		
			95	99516	9							
				9020	0	D	2	04133	72028	2	B	

In all these cases, both in the powers of numbers, and in Logarithms, Tangents and Secants, it will be necessary to include in the

work several more numbers than those between which we interpolate; or we shall not be able to obtain the last differences. Thus, in the example given above, we must take in one direction the numbers 2110 and 2105; and in the other direction 2130 and 2135. But in the case of Sines, if the sines of three equidifferent arcs are given, all the differences, even of the highest order, can be found by the rule of proportion, if required. For the Sines and their Second, Fourth, Sixth, and Eighth differences are always proportional; and the First, Third, Fifth, and Seventh differences are also always proportional. Thus, as the Second differences are themselves proportional to the corresponding Sines; as are also the Fourth, Sixth, &c. differences; so the First, Third, Fifth and Seventh differences are proportional to the cosines of the arcs which are the arithmetic means of the given arcs.

But I feel I have been carried away by these considerations into a longer digression than is warranted by the laws of homogeneous quantities. If you wish to compute another Thousand Logarithms to be added to those I have calculated (suppose the twenty-first Thousand) you must take the fifth part of that number from which you are to begin. The first number will then be 20,000, the fifth part of which is 4000. To the Logarithms of this number and of the next two hundred numbers, add the Logarithm of 5; then the sums will be the Logarithms of each fifth number through the whole Thousand: namely, of 20000, 20005, 20010, 20015, &c. Now their first differences are the same as those of the above two hundred Logarithms; and are found* in the fifth Thousand of my tables. From these differences are to be found the second differences. The second differences will likewise give the third. The fourth differences are however very small, so that we may safely neglect them. Then multiply the first, second and third differences into two [·2], four [·04], eight [·008]. The products will be the mean differences that are to be inserted in their respective places, having first cut off one figure from the second, and two from the third differences. But the first differences are to be kept out of their places until they have been corrected by subtracting the third differences: all the rest are to be obtained by *addition.*

This method of interpolating four Logarithms between two given ones, may be called *Quintisection,* because from one interval five are to be made. General rules can also be given for *Trisection,*

* Briggs's Tables of Logarithms contain, not only the logarithms to 14 decimal places, but also the differences between successive logarithms.

and *Septisection*; but of all these, Quintisection is the best, whether we regard the length or the facility of the computation. Nevertheless it will be worth while to give in a few words the method of Trisection. Take as before the first, second, third, &c., differences of the given quantities. Then divide the first differences by 3, the second by 9, the third by 27, the fourth by 81, and so on; the divisors increasing in triple ratio: and the quotients will be the first, second, third, fourth, &c., *mean* differences. These mean differences are, as before, to be diminished in all cases except in the case of Sines; and then the corrected differences are to be put into their proper places: and, commencing with the differences of the highest order, which are supposed to be the smallest, all the work is to be done as before by addition.

The annexed Table shows how much is to be subtracted from each difference:

1 (12) 1 (11)				
1 (10) 1 (9)	$3\frac{1}{3}$ (12) 3 (11)			
1 (8) 1 (7)	$2\frac{2}{3}$ (10) $2\frac{1}{3}$ (9)	$3\frac{1}{9}$ (12) $2\frac{3}{9}$ (11)		
1 (6) 1 (5)	2 (8) $1\frac{2}{3}$ (7)	$1\frac{6}{9}$ (10) $1\frac{1}{9}$ (9)	$\frac{20}{27}$ (12) $\frac{10}{27}$ (11)	
1 1	$1\frac{1}{3}$ (6) 1 (5)	$\frac{6}{9}$ (8) $\frac{3}{9}$ (7)	$\frac{4}{27}$ (10) $\frac{1}{27}$ (9)	$\frac{1}{81}$ (12)
1 1	$\frac{2}{3}$ (4) $\frac{1}{3}$ (3)	$\frac{1}{9}$ (6)		
A	B	C	D	E

From the first mean difference we must take $\frac{1}{3}$ of the third corrected difference.

From the fourth mean difference we must take $\frac{4}{3}$ of the sixth, $\frac{2}{3}$ of the eighth, $\frac{4}{27}$ of the tenth, $\frac{1}{81}$ of the twelfth corrected differences.

The other Sections, named after the even numbers, as Bisection, Quadrisection, &c., are more difficult. This we also experience in finding the *chords* of circular arcs: for whilst the sections named after the odd numbers show the required chords themselves at one operation; the others, named after the even numbers, develope, not the chords, but only their squares.

Here is an example of Trisection in the Fourth powers.

Mean Differences found by division of the given differences.				Given Differences.				Fourth Powers.	Numbers.
4th by 81.	3d by 27.	2nd by 9.	1st by 3.	4th.	3d.	2nd.	1st.		
								256	4
		$33\frac{45}{81}$	$223\frac{2}{3}$	24		302		625	5
	d. $4\frac{72}{81}$				132		671		
$\frac{24}{81}$		$48\frac{18}{81}$		24		434		1296	6
	d. $5\frac{63}{81}$		$368\frac{1}{3}$		156		1105		
		$65\frac{45}{81}$		24		590		2401	7
								4096	8

The third and fourth mean differences cannot be corrected.

If $\frac{2}{3}$ of the fourth difference be subtracted from the second mean differences, the remainders will be the second corrected differences C.

If $\frac{1}{3}$ of the third difference be subtracted from the first mean differences, the remainders will be the first corrected differences B, as appears from the table X, *ante.*

4th Diffces.	3rd Differences.	2nd Differences.		1st Differences.	4th Powers.	Numbers.
$\frac{24}{81}$		C. $33\frac{29}{81}$			625. A	5
	$4\frac{48}{81}$			$184\frac{7}{81}$		
		$37\frac{77}{81}$			$809\frac{7}{81}$	$5\frac{1}{3}$
	$4\frac{72}{81}$		B	$222\frac{3}{81}$		
	D	$42\frac{68}{81}$			$1031\frac{10}{81}$	$5\frac{2}{3}$
	$5\frac{15}{81}$			$264\frac{71}{81}$		
$\frac{24}{81}$		C. $48\frac{2}{81}$			1296. A	6
	$5\frac{39}{81}$			$312\frac{73}{81}$		
		$53\frac{41}{81}$			$1608\frac{73}{81}$	$6\frac{1}{3}$
	$5\frac{63}{81}$			$366\frac{33}{81}$		
	D	$59\frac{23}{81}$	B		$1975\frac{25}{81}$	$6\frac{2}{3}$
	$6\frac{6}{81}$			$425\frac{56}{81}$		
$\frac{24}{81}$		C. $65\frac{29}{81}$			2401. A	7

We next give a translation of the paper by Legendre in the additions to the *Connaissance des Tems* for 1817, (mentioned by M. Maurice), in which he demonstrates the reasons of the rules laid down by Briggs.

We are indebted to Henry Briggs, Professor of Geometry at Oxford, for two fundamental works, the *Arithmetica Logarithmica* published at London in 1624, and the *Trigonometria Britannica*

published at Gouda in 1633. Each of these works is prefaced by a treatise in which the author has explained, with all necessary details, the various methods employed by him in constructing his Tables. These methods are principally his own invention, and prove him to have been quite familiar with the theory of differences, although he was not acquainted with the general formula for interpolating intermediate values of a function in a series of values corresponding to equidistant values of the argument.

Briggs supplied the place of this formula by a very remarkable method, which may be called the *Method of Quintisection*, by means of which, if a series of equidistant values are given, we may interpolate between any two adjacent values, four others, so that the total number of the terms of the series shall be five times as many as before. In this manner, Briggs extended by successive steps the various tables he wished to construct, until the scheme he had proposed to himself was completed. He does not however give any demonstration of this method, but simply explains the process in the clearest manner, giving numerous illustrations in both his above-mentioned works.

It does not appear that this method has ever attracted much attention, or that any one has tried to demonstrate it. If however we consider that these very works of Briggs's have been the foundation of all or nearly all the Trigonometrical Tables hitherto published, that it is only by means of the Tables they contain that we can, without great labour, find the logarithm of a number, or of a sine, to 14 places of decimals, and a natural sine to 15 places; it will probably not be thought surprising that I have examined with some interest one of the principal bases on which those two great works have been constructed.

I will now proceed with the demonstration at which I have arrived. It is not so simple as I could have wished; but it may perhaps lead to the discovery by some other mathematician of a demonstration more akin to that which the author himself must have discovered, although he neglected to publish it.

Given a series of values of a function, $a, a', a'', a''', \ldots$; corresponding to the values of the argument $0, 1, 2, 3, \ldots$; and such, that their first, second, third, &c., differences constantly diminish and at last become so small that they may be neglected; it is required to interpolate four equidistant values between any adjacent two of the given values, so that the terms of the resulting series shall correspond to arguments differing by $\frac{1}{5}$.

By means of this interpolation which quintuples the number of terms, we shall have the new series:

$$y,\ y',\ y'',\ y''',\ y^{\text{IV}},\ y^{\text{V}},\ y^{\text{VI}},\ y^{\text{VII}},\ y^{\text{VIII}},\ y^{\text{IX}},\ y^{\text{X}},\ldots;$$

corresponding to arguments

$$0,\ \frac{1}{5},\ \frac{2}{5},\ \frac{3}{5},\ \frac{4}{5},\ 1,\ \frac{6}{5},\ \frac{7}{5},\ \frac{8}{5},\ \frac{9}{5},\ 2,\ldots;$$

and the terms $y, y^{\text{V}}, y^{\text{X}}, \ldots$, corresponding to the integral arguments 0, 1, 2, . . . , will be the same as the given values $a, a', a'', \ldots$.

I now remark, that according to the known formula for interpolation, we have

$$y'-y=\delta y=a[(1+\delta)^{\frac{1}{5}}-1]$$

provided that after expanding $(1+\delta)^{\frac{1}{5}}$, we change the products $a\delta, a\delta^2, a\delta^3, \ldots$, into successive differences, $\delta a, \delta^2 a, \delta^3 a, \ldots$. Putting $(1+\delta)^{\frac{1}{5}}=\omega$, we shall have

$$\delta y=a(\omega-1).$$

Under the same supposition we shall have, $y'=a\omega$, $y''=a\omega^2$, $y'''=a\omega^3, \ldots$. We may thus form the following table, which contains the symbolical values of the terms $y', y'', y''', \ldots$, and of their successive differences.

Terms.	1st Differences.	2nd Differences.	3d Differences.	4th Differences.	&c.
$y=a$	$\delta y=a(\omega-1)$	$\delta^2 y=a(\omega-1)^2$	$\delta^3 y=a(\omega-1)^3$	$\delta^4 y=a(\omega-1)^4$	&c.
$y'=a\omega$	$\delta y'=a\omega(\omega-1)$	$\delta^2 y'=a\omega(\omega-1)^2$	$\delta^3 y'=a\omega(\omega-1)^3$	$\delta^4 y'=a\omega(\omega-1)^4$	&c.
$y''=a\omega^2$	$\delta y''=a\omega^2(\omega-1)$	$\delta^2 y''=a\omega^2(\omega-1)^2$	$\delta^3 y''=a\omega^2(\omega-1)^3$	$\delta^4 y''=a\omega^2(\omega-1)^4$	&c.
$y'''=a\omega^3$	$\delta y'''=a\omega^3(\omega-1)$	$\delta^2 y'''=a\omega^3(\omega-1)^2$	$\delta^3 y'''=a\omega^3(\omega-1)^3$	$\delta^4 y'''=a\omega^2(\omega-1)^4$	&c.
$y^{\text{IV}}=a\omega^4$	$\delta y^{\text{IV}}=a\omega^4(\omega-1)$	$\delta^2 y^{\text{IV}}=a\omega^4(\omega-1)^2$	$\delta^3 y^{\text{IV}}=a\omega^4(\omega-1)^3$	$\delta^4 y^{\text{IV}}=a\omega^4(\omega-1)^4$	&c.
$y^{\text{V}}=a\omega^5$	$\delta y^{\text{V}}=a\omega^5(\omega-1)$	$\delta^2 y^{\text{V}}=a\omega^5(\omega-1)^2$	$\delta^3 y^{\text{V}}=a\omega^5(\omega-1)^3$	$\delta^4 y^{\text{V}}=a\omega^5(\omega-1)^4$	&c.
&c.	&c.	&c.	&c.	&c.	&c.

The law of these expressions is evident, and we shall have, generally, whatever may be the values of m and n,

$$\delta^m y^n=a\omega^n(\omega-1)^m;$$

where it is supposed that after having substituted the value $\omega=(1+\delta)^{\frac{1}{5}}$, and expanded the second member by powers of δ, each term as $a\delta^n$, is to be replaced by the difference $\delta^n a$.

All is known when the expansions are completed, but the process is long and troublesome. There is a simpler way of forming

the different values y', y'', , . . by seeking for the relations that may exist between a finite number of them.

With this object, I note that the quantities which may be found directly from the given values are

$$\delta a = a(\omega^5-1),\ \delta^2 a = a(\omega^5-1)^2,\ \delta^3 a = a(\omega^5-1)^3,\ \&c.$$

I write the first under the form

$$\delta a = a(\omega-1)(\omega^4+\omega^3+\omega^2+\omega+1)$$

and making $(\omega-1)^2=\omega Z$, which gives

$$\omega^2+1=\omega(Z+2),\ \omega^4+1=\omega^2(Z^2+4Z+2),$$

I deduce $$\delta a = a\omega^2(\omega-1)(Z^2+5Z+5),$$

or $$\frac{\delta a}{5} = a\omega^2(\omega-1)\left(1+Z+\frac{1}{5}Z^2\right).$$

But if in the table of Differences we introduce the quantity Z, so as not to have any power of $\omega-1$ above the first, we shall have

$\delta y\ = a\ \ (\omega-1)$	$\delta^2 y\ = a\omega\ Z$	$\delta^3 y\ = a\omega\ Z(\omega-1)$	$\delta^4 y\ = a\omega^2 Z^2$
$\delta y' = a\omega\ (\omega-1)$	$\delta^2 y' = a\omega^2 Z$	$\delta^3 y' = a\omega^2 Z(\omega-1)$	$\delta^4 y' = a\omega^3 Z^2$
$\delta y'' = a\omega^2(\omega-1)$	$\delta^2 y'' = a\omega^3 Z$	$\delta^3 y'' = a\omega^3 Z(\omega-1)$	$\delta^4 y'' = a\omega^4 Z^2$
$\delta y''' = a\varpi^3(\omega-1)$	$\delta^2 y''' = a\omega^4 Z$	$\delta^3 y''' = a\omega^4 Z(\omega-1)$	$\delta^4 y''' = a\omega^5 Z^2$
$\delta y^{\text{iv}} = a\omega^4(\omega-1)$	$\delta^2 y^{\text{iv}} = a\omega^5 Z$	$\delta^3 y^{\text{iv}} = a\omega^5 Z(\omega-1)$	$\delta^4 y^{\text{iv}} = a\omega^6 Z^2$
&c.	&c.	&c.	&c.

From this we see that we may write the preceding equation thus:

$$\frac{\delta a}{5} = \delta y'' + \delta^3 y' + \frac{1}{5}\delta^5 y.$$

In the same way the equation $\delta^2 a = a(\omega^5-1)^2$ will become

$$\frac{\delta^2 a}{25} = a\omega^4(\omega-1)^2\left(1+Z+\frac{1}{5}Z^2\right)^2 = a\omega^5 Z\left(1+Z+\frac{1}{5}Z^2\right)^2$$

and expanding the second member, we get

$$\frac{\delta^2 a}{25} = \delta^2 y^{\text{iv}} + 2\delta^4 y''' + \frac{7}{5}\delta^6 y'' + \frac{2}{5}\delta^8 y' + \frac{1}{25}\delta^{10} y.$$

Similarly, by expanding the cube of the trinomial $1+Z+\frac{1}{5}Z^2$, we shall have the equation

$$\frac{\delta^3 a}{125} = \delta^3 y^{\text{vi}} + 3\delta^5 y\ + 3{\cdot}6\delta^7 y^{\text{iv}} + 2{\cdot}2\delta^9 y''' + {\cdot}72\delta^{11} y'' + {\cdot}12\delta^{13} y' + {\cdot}008\delta^{15} y.$$

These three equations, and those which we should form in the same way for the values of $\frac{\delta^4 a}{5^4}$, $\frac{\delta^5 a}{5^5}$, &c., express the same thing as the Table given by Briggs in the *Arith. Logarith. Ed. Lond.* p. 29 (see above, p. 79); and in the *Trigon. Brit.* p. 38.

The quantities $\frac{\delta a}{5}$, $\frac{\delta^2 a}{5^2}$, $\frac{\delta^3 a}{5^3}$, . . . , are what Briggs calls *mean differences;* they give a first approximation to the differences $\delta y''$, $\delta^2 y^{\text{IV}}$, $\delta^3 y^{\text{VI}}$, . . . but these values require to be corrected by means of the following terms. But, from the nature of the case, the successive differences δy, $\delta^2 y$, $\delta^3 y$, . . . must diminish very rapidly, and it will therefore not be necessary to go beyond that order of differences which may be safely neglected in the series a, a', a'', . . and much more in the series y, y', y'', . . . We may therefore suppose the two last differences of the series δy, $\delta^2 y$, $\delta^3 y$, . . . equal to the two mean differences deduced from the two last terms of the series δa, $\delta a'$, $\delta a''$, . . . We must then correct the other differences, beginning with those of the highest order and ending with the differences $\delta^3 y^{\text{VI}}$, $\delta^2 y^{\text{IV}}$, $\delta y''$. We shall thus have the corrected values of these last differences, and by means of these and the preceding ones we can complete by addition the columns of the differences, and lastly we can form the column of values y, y', y'', y''', . . . This is in fact the method of Briggs, which it was our object to demonstrate, and which, although somewhat complicated in appearance, is rendered perfectly clear by the examples the author has given.

Our readers will now be in a position to judge for themselves of the justice of M. Maurice's strictures on Briggs's method. They will see that there is no possibility of confusion in consequence of the differences of various orders having relation to different terms in the series of given values, provided that the differences are written in the usual way—each difference on a line half way between the two values from which it is obtained. They will also see that M. Maurice in asserting that the values in the table on p. 81 are obtained by *subtraction*, and not by *addition*, as stated by Briggs himself, (see p. 12 of this volume,) overlooked the circumstance that a computer would naturally form the successive values by addition, *commencing from the bottom*; instead of beginning at the top and using subtraction. There can be no doubt, we believe, that Briggs's description of his method will be found perfectly clear by any computer wishing to apply it in practice.—ED. *J. I. A.*